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Trapping Without Traps by Correlated Random Walks

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Correlated random walk of particles in the infinite cluster of percolating lattices in two dimensions is investigated. For infinitely strong forward correlations (no change of direction except at the boundaries) trapping of the particles in small regions of the infinite cluster is observed.

KEY WORDS: Percolation; diffusion; fractals; correlated random walks; number of visited sites.

The detailed understanding of transport of particles or energy in disordered systems in condensed phases has been a problem of prime interest recently.(1–3) It has been visualized from quite different points of view, such as, for example, from studies of dynamics of a lattice gas or the noncrystalline character of a solid in materials research, etc. A systematic approach for the study of disorder in solids has been provided by the picture of fractal structures. The scaling laws that were found to hold in these structures have certainly improved our understanding of transport in these complex systems. We have explicit expressions for the main features of random walk on a fractal, as, for example, it is given by a percolation cluster exactly at the critical threshold. As more complicated systems were investigated experimentally(4,5) more sophisticated models were necessary for their interpretation. One such model is that of correlated random walk where the moving particle retains the memory of its previous move regarding its particular direction. A forward correlation is introduced when the probability for a move in the same direction as the previous one is higher than in the

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other directions. For strong forward correlation the particle may move in the same direction for long times before scattering to some other direction.

We are interested in the behavior of correlated walks in models with randomly blocked sites, especially at the percolation threshold.(6) There it has been shown that correlations of moderate strength do not change the universal behavior of the mean square displacement.(7) To characterize the influence of correlations, the efficiency may be introduced, which is simply defined as the ratio of the number of distinct sites visited over time. The principal results of the past work were that correlated motion enhances the efficiency of a random walk on a perfect lattice, while it decreases this efficiency in highly disordered systems.(5,8,9) The same trend appears for the mean square displacement, i.e., it increases with increasing correlation in perfect lattices, but decreases in highly disordered lattices, as compared to normal random walks. Of course, if the concentration of randomly blocked sites is increased beyond a critical point, all transport becomes restricted to small finite clusters regardless of the degree of correlation present. These trends have been qualitatively explained in terms of the spatial constraints imposed on the transport. In the case of perfect lattices retention of directional memory helps the particle to move further away from the origin. For the percolation cluster we have a very ramified structure where the effect of correlation results in producing a back-and-forth motion, while the simple random walk is more efficient in escaping from bottlenecks, dead ends, etc.

In this note we report an interesting observation for the case of the infinitely strongly correlated motion on a fractal, the two-dimensional percolation cluster at the critical point. Namely, we observe that for this case the particle is fully "trapped" in some small-size region of the infinite cluster and stays there "forever," i.e., independent of time. This is shown both by the behavior of S_n, the number of distinct sites visited, and of R^2, the mean square displacement, respectively. If correlation is large but not infinite, this trapping is not observed,(9) but a time dependence of these quantities appears.

Our previous studies(5) concentrated on a rather limited range in the values of the correlation parameter. The main unexpected result had been the decrease in the transport efficiency for highly disordered lattices. Here we deal exclusively with the limiting case of infinitely long correlation ($p_f = 1.00$) having as a consequence the effective "trapping," which has not been observed before.

The quantitative measure of correlation in our model is given by the probability p_f, which is the probability for a step in the same direction as the preceding one, while $1 - p_f$ is the probability for steps in all other $(z-1)$ directions combined (z is the coordination number). Thus p_f is in the range $1/z \leqslant p_f \leqslant 1$. The case $p_f = 1/z$ refers to the uncorrelated case, while $p_f = 1.00$ gives the case of infinite correlation, the topic of our note. We investigate the infinitely correlated walk by numerical simulation. Analytical treatments have been made for perfect lattices[8] ($p_f < 1$), or in the form of effective medium theory,[10] which applies away from the percolation threshold. Our algorithm proceeds as follows: A binary lattice (open–closed sites) is constructed exactly at the critical percolation threshold p_c. We use $p_c = 0.593$ and employ the cluster multiple labeling technique[11] to construct the infinite cluster. A particle is placed at random on this cluster and performs a random walk with the correlation parameter $p_f = 1.00$. This means that the particle initially chooses a direction of motion and moves on a straight line until it reaches the boundary of the cluster. At this point it simply loses memory and chooses a new direction, and again moves on a straight line without any interruption until it hits the boundary again. This process is repeated for 2.5×10^5 steps.

Figure 1 shows the behavior of S_n as a function of time (number of steps) for this model of infinite correlation ($p_f = 1.00$) on a two-dimensional percolation cluster at $p = p_c = 0.593$. We observe that very early (below 10,000 steps) a constant value of S_n is reached which is about $S = 60$ sites. In no part of this curve (apart from the constant value) do we observe any power-law behavior, as we would expect for uncorrelated random walk, for which S_n obeys the form

$$S_n \sim n^{d_s/2} \tag{1}$$

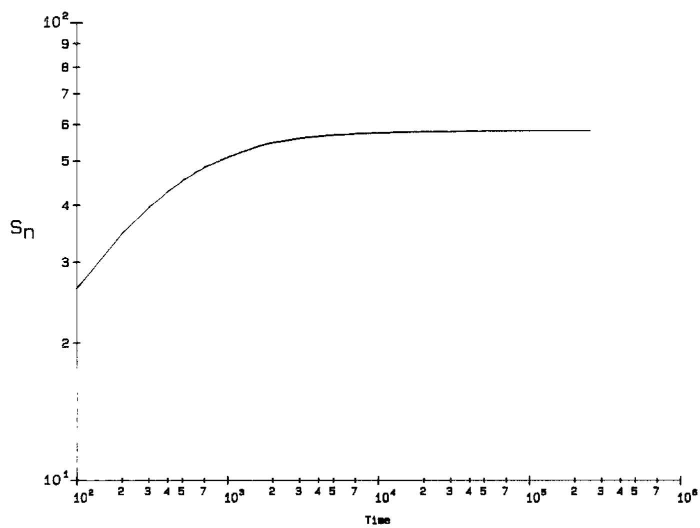

Fig. 1. Plot of S_n as a function of time in log–log scale for the limit of infinite correlation, $p_f = 1.00$. Lattice size is 500×500 sites. These results are averages of 2000 realizations of the percolating cluster.

where d_s is the spectral (fracton) dimension of the process. This is obviously a consequence of the fact that the particle is trapped in some region of the cluster as these "wall-to-wall" distances are traversed. Once it is placed in the cluster the particle cannot escape from a restricted region, even though the cluster is infinite. It simply cannot find the narrow way out of this region due to this form of scattering, which only takes place at the "walls." Once p_f becomes smaller than 1.00 then the situation drastically changes and Eq. (1) is obeyed asymptotically.

The question arises as to how large in size are these regions of trapping. In Fig. 2 we present the complete distribution of S_n at the long-time limit. We observe that this distribution has a maximum much below $\langle S \rangle$, at about $S = 20$ sites, and a long tail at its right side. The distribution of quantities due to disorder is an important concept for a disordered system,[12] which was investigated recently for random chains.[13] Here we have another system where the typical behavior of a quantity does not coincide with its mean value. To determine the precise form of the distribution we plotted it in Fig. 3 in a linear-log scale. The figure suggests an exponential form of the distribution, although there seem to be small deviations. We have also plotted the data in a log-log scale; clearly there is no simple power-law decay of the distribution present. An exponential form of the distribution,

$$p(S) \sim \exp(-S/\sigma) \tag{2}$$

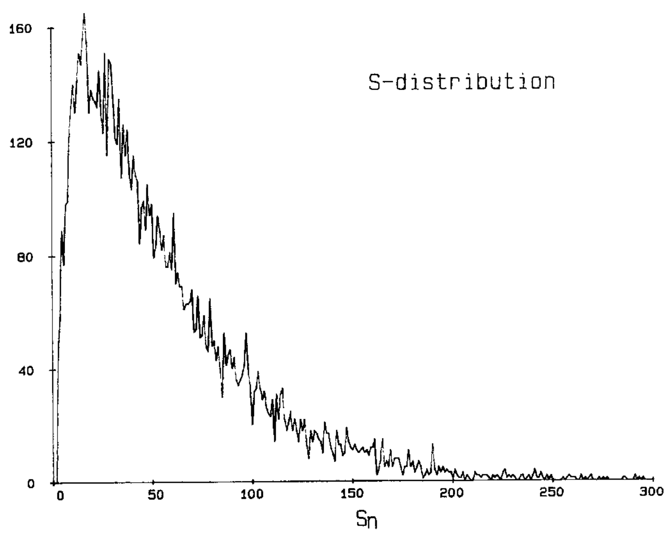

Fig. 2. Complete distribution of S_n for $n = 5 \times 10^4$ steps. 10,000 realizations were utilized for this plot, and the interval $\Delta S = 1$ site.

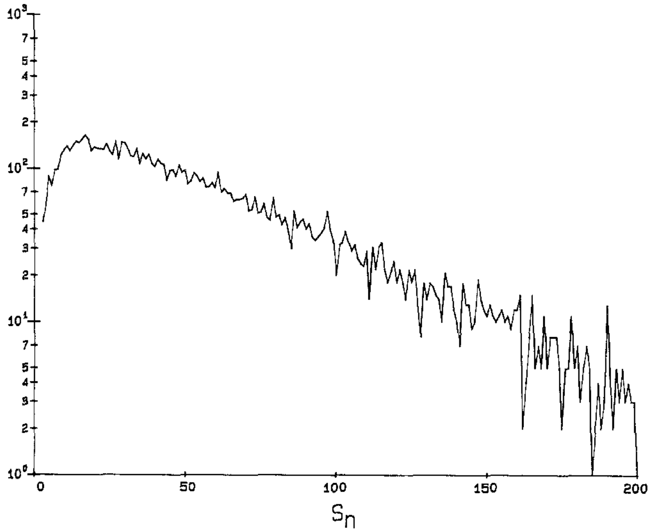

Fig. 3. Same plot as in Fig. 2, but in linear–log scale.

would entail that all moments of the number of sites visited exist, in contrast to a power-law decay, which leads to divergent moments. Of course, a more complete examination of the distribution is desirable and it is currently being worked out.

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