Analytical expression of number of discrete sites visited

The analytical expression for S_N in the asymptotic limit of $N \to \infty$, where N is the number of steps, was given by Montroll and Weiss for all three dimensionalities.

In the case of a 1-D lattice, S_N follows a $N^{1/2}$ power law, in the 2-D lattice case its behaviour is approximately linear (with a logarithmic term in the denominator) and in the 3-D lattice case it grows linearly with N:

1-D $$S_N \cong \left(\frac{8N}{\pi}\right)^{\frac{1}{2}}, \quad N \to \infty \tag{1}$$

2-D $$S_N \cong \frac{\pi N}{\ln(N)}, \quad N \to \infty \tag{2}$$

3-D $$S_N \cong 0.66\,N, \quad N \to \infty \tag{3}$$

Eq. (1), (2), and (3) describe the asymptotic behaviour. The following terms increase the accuracy of the early-time behaviour.

1-D $$S_N \cong \left(\frac{8N}{\pi}\right)^{\frac{1}{2}} \left(1 + \frac{1}{4N} - \frac{3}{64N^2} + \cdots\right), \quad N \to \infty \tag{4}$$

2-D $$S_N \cong \frac{N\pi}{ln(8N)} \sum_{j=0}^{\infty} \frac{c_j}{(ln8N)^j} [1 + O(N)], \quad N \to \infty \tag{5}$$

3-D $$S_N \cong 0.65946267N + 0.573921N^{1/2} + 0.449530 + 0.40732N^{-1/2} + \cdots, \quad N \to \infty \tag{6}$$

The c_j coefficients in (4), for j up to 20, are shown in the following table:

j	c_j	j	c_j
0	1.000000	11	103.344
1	0.422784	12	641.144
2	-0.466187	13	1279.49
3	-1.146547	14	-13.91
4	-0.589260	15	-8206.5
5	2.117429	16	-26647.0
6	5.77676	17	-32844.0
7	4.05382	18	76848.0
8	-14.5490	19	513400.0
9	-52.8339	20	1275000.0
10	-63.6704		