Cluster Multiple Labelling Technique (CMLT)

The cluster distribution in the Percolation Model is performed using the so-called Cluster-Multiple-Labelling-Technique (CMLT), which was developed in the paper by J. Hoshen and R. Kopelman, Phys. Rev. B14, 3438(1976). The principal idea in the CMLT is that when sweeping the lattice, when one comes to the case of cluster coalescence, no sites need to be re-indexed, once a site has been indexed. All sites retain their original index throughout the sweep of the entire lattice. The difference from the "brute force" method is a new index processing strategy that enables us, in the case of a square lattice topology where every site has two different neighbours, to check each site only once and no second sweep is needed. The sizing of each cluster is also done during this single sweep. The algorithm is given below:

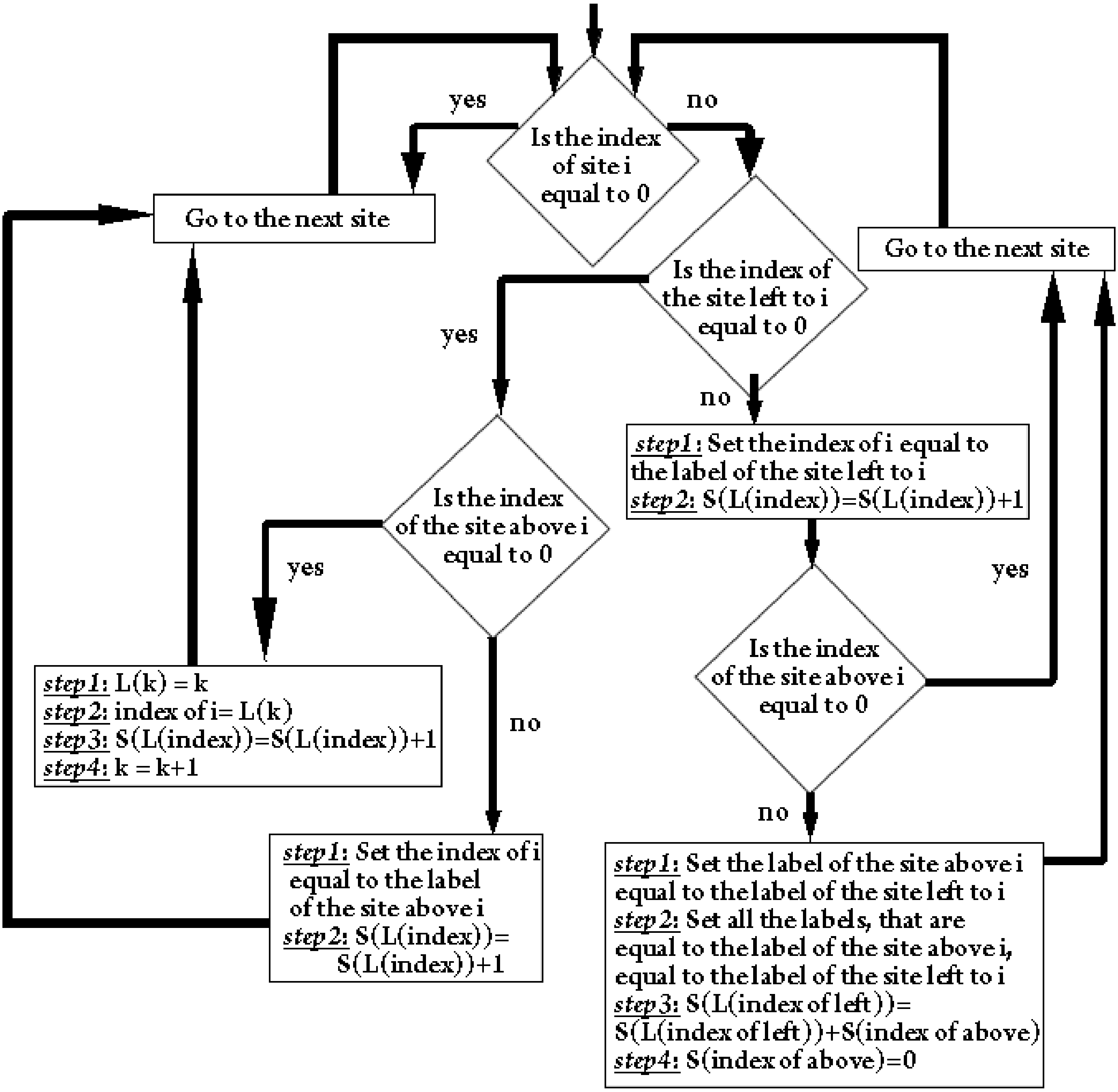

Figure 1: Flow chart for the single sweep site labeling strategy.

Before the sweep, occupied (open) sites are labelled 1 and empty (closed) sites are labelled 0. All labelling is done column by column, starting at the top of the left most column. We label each open site using a one-dimensional array **L** as follows: If both sites above and to the left of an open site i are closed, then i takes, the current index of **L**, which in the beginning is set equal to 1. If only one of these two sites is open, then i takes the label of the open site. If both these two sites are open, then the labels of site i and of the site above i simultaneously take the label of the site to the left of i. This is all carried out using the one-dimensional array **L**, where the index k (k=1, 2, 3,… etc) of site i is the index which satisfies the equation **L**(k) =k. The label of site I which will be stored in **L** is determined as follows: If the original index of site (i-1) is k_1 and the label of site (i-1) is k_2, then, for site i, we set **L**(k_1)=k_2 and **L**(k_2)=k_2 so that k_2 is now the stored label for this site. We also use a one-dimensional array **S**, to store the size of each cluster. Each time we label a site, we increase the corresponding entry of **S** by one. When both the sites above and to the left of i are open and they have different labels, then there is a merging of two clusters. We then add the size of the cluster whose label is equal to the index of the site above i, to the size of that whose label is equal to that of the site to the left of i. The size of the first cluster is then zeroed out. The flowchart for this algorithm is given in Figure 1.

Once the sweep is finished, all sites are accurately labelled and all the sizes of the clusters are evaluated, therefore **there is no actual need of a second sweep**. However, we can perform a second sweep to assign the indexes of all open sites according to their labels, in the case we need to visualise the lattice with all the clusters correctly re-indexed, for instance if we want to check the validity of our implementation of the algorithm. The flowchart for this second sweep is given in Fig. 3.

To make this process better understood we work out an example of index and label processing for a 20x20 square lattice, as shown in Fig. 2 The assigned p is p=0.56. Part (a) shows the lattice before the sweep, with 1 and 0 for open and closed sites, respectively. In part (b), we begin by labelling the open sites of the first column. Following the algorithm in Fig. 1, the first label to be used for the first cluster is 1, hence **L**(1)=1 and the index of the first open site, which is the first site of this first cluster, is: index=**L**(1)=1. The size of this first cluster with index 1 is **S**(1)=1. The second and third open sites are also indexed 1, which is the label of the site above them. Now we have **S**(1)=3. The fourth site is the first site of the second cluster which is to be labelled 2, and so its index is **L**(2)=2, **S**(2)=1 and so on. Moving on to the second column, the first site takes the label of its left site, which is **L**(1)=1 and **S**(1)=4, the next open site is indexed 5, as **L**(5)= 5, with **S**(5)=1. When we reach the 13^{th} site in the second column, a merging of two clusters occurs. First, we set the index of this site equal to the label of its left site, so index=**L**(3)=3 and **S**(3)=3. Then, we set all the labels that are equal to the label of the site above it equal to the label of the site left to it, so **L**(2)=3 and **S**(3)=13, **S**(2)=0. We finish labelling the rest of the second column in this manner. The first site in the third column takes the index 6, as **L**(6)=6, with **S**(6)=1. The next one is indexed 1, because the label of its left site is **L**(1)=1 and **S**(1)=5, while there is another merging of clusters, so **L**(6)=**L**(1)=1 and **S**(1)=6, **S**(6)=0. The index of the fourth site in the third column is **L**(5)=5, with **S**(5)=2, and since there is another merging **L**(1)=**L**(5)=5 and **S**(5)=**S**(5)+**S**(1)=2+7=9. When we reach the fourth column, the first open site takes the index 5, because now the label of its left site is **L**(6)=**L**(1)=**L**(5)=5, with **S**(5)=10, etc. Using this strategy, we label all open sites in the lattice. Although there is no actual need to do so, we can perform a second sweep, upon which we re-index all open sites according to their labels. The final form of the lattice with all the different clusters correctly re-indexed, after the second sweep is finished, is shown in part (c). As we can easily see, the percolating cluster consists of 83 lattice sites and its index is 50, the second largest cluster has a size of 71 and a lattice index of 22 and so on.

1	0	1	1	0	0	0	1	1	0	1	0	1	0	0	0	1	1	0	1
1	1	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	0	1	1
1	0	1	0	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	0
0	1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1	0	1	1
0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	0	1	0	1	1
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	0	1	1	1	0	0	1	0
1	1	1	0	1	1	1	1	1	0	1	0	1	0	0	0	0	0	1	1
1	1	0	0	1	1	0	0	0	0	1	1	0	1	0	0	0	0	0	1
1	0	1	0	0	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1
1	1	0	1	1	0	1	1	1	1	0	1	0	0	1	1	1	0	0	0
0	1	1	1	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1
1	1	1	0	0	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1
1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	0	0	1
0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	0	1
0	1	0	1	1	1	1	0	1	1	0	1	0	1	0	1	0	1	1	1
1	0	1	1	0	0	0	1	1	1	1	0	0	0	1	1	0	1	1	1
1	1	1	0	0	0	1	0	1	0	1	0	0	1	0	0	1	1	0	1
0	1	1	1	0	0	1	0	0	1	0	0	0	1	0	1	1	0	1	0
0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

Part (a)

1	0	6	5	0	0	0	18	18	0	26	0	33	0	0	0	43	31	0	52
1	1	1	5	0	13	0	0	0	24	0	30	11	31	31	31	31	0	49	31
1	0	1	0	10	5	0	0	0	24	11	11	11	31	31	31	31	0	49	0
0	5	5	5	5	0	0	19	11	11	11	0	11	31	31	31	31	0	49	31
0	0	0	0	5	5	11	11	0	11	0	31	31	0	0	0	31	0	49	31
0	0	0	0	0	5	11	11	0	0	0	0	0	0	0	0	31	31	31	0
2	2	3	0	0	5	11	11	11	0	27	27	0	37	37	37	0	0	31	0
2	2	3	0	11	11	11	11	11	0	27	0	34	0	0	0	0	0	31	31
2	2	0	0	11	11	0	0	0	0	27	27	0	38	0	0	0	0	0	31
2	0	7	0	0	11	0	20	14	14	0	0	0	38	38	0	0	0	50	50
2	2	0	9	4	0	15	14	14	14	0	32	0	0	38	38	38	0	0	0
0	2	3	3	4	0	15	0	14	0	28	0	35	0	38	38	0	0	0	53
3	3	3	0	0	14	14	0	0	0	0	0	35	0	0	38	0	46	0	53
3	3	3	3	4	0	0	21	4	0	29	22	22	22	22	0	44	0	0	53
0	3	0	3	4	0	16	4	4	22	22	22	22	22	22	0	0	47	0	53
0	3	0	3	4	4	4	0	4	22	0	22	0	22	0	41	0	47	42	42
4	0	8	4	0	0	0	22	22	22	22	0	0	0	40	40	0	47	42	42
4	4	4	0	0	0	17	0	22	0	22	0	0	39	0	0	45	42	0	42
0	4	4	4	0	0	17	0	0	25	0	0	0	39	0	42	42	0	51	0
0	0	4	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (b)

50	0	50	50	0	0	0	18	18	0	26	0	50	0	0	0	50	50	0	50
50	50	50	50	0	50	0	0	0	50	0	50	50	50	50	50	50	0	50	50
50	0	50	0	50	50	0	0	0	50	50	50	50	50	50	50	50	0	50	0
0	50	50	50	50	0	0	50	50	50	50	0	50	50	50	50	50	0	50	50
0	0	0	0	50	50	50	50	0	50	0	50	50	0	0	0	50	0	50	50
0	0	0	0	0	50	50	50	0	0	0	0	0	0	0	0	50	50	50	0
22	22	22	0	0	50	50	50	50	0	27	27	0	37	37	37	0	0	50	0
22	22	22	0	50	50	50	50	50	0	27	0	34	0	0	0	0	0	50	50
22	22	0	0	50	50	0	0	0	0	27	27	0	38	0	0	0	0	0	50
22	0	7	0	0	50	0	14	14	14	0	0	0	38	38	0	0	0	50	50
22	22	0	22	22	0	14	14	14	14	0	32	0	0	38	38	38	0	0	0
0	22	22	22	22	0	14	0	14	0	28	0	22	0	38	38	0	0	0	42
22	22	22	0	0	14	14	0	0	0	0	0	22	0	0	38	0	46	0	42
22	22	22	22	22	0	0	22	22	0	22	22	22	22	22	0	44	0	0	42
0	22	0	22	22	0	22	22	22	22	22	22	22	22	22	0	0	42	0	42
0	22	0	22	22	22	22	0	22	22	0	22	0	22	0	40	0	42	42	42
22	0	22	22	0	0	0	22	22	22	22	0	0	0	40	40	0	42	42	42
22	22	22	0	0	0	17	0	22	0	22	0	0	36	0	0	42	42	0	42
0	22	22	22	0	0	17	0	0	25	0	0	0	36	0	42	42	0	48	0
0	0	22	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (c)

Figure 2 : An example of a cluster distribution for a 20x20 lattice. Part (a): A binary, substitution all random square lattice with an assigned p=0.56. Part (b): Index assignment after the first sweep. Part (c): Index assignment after the second sweep.

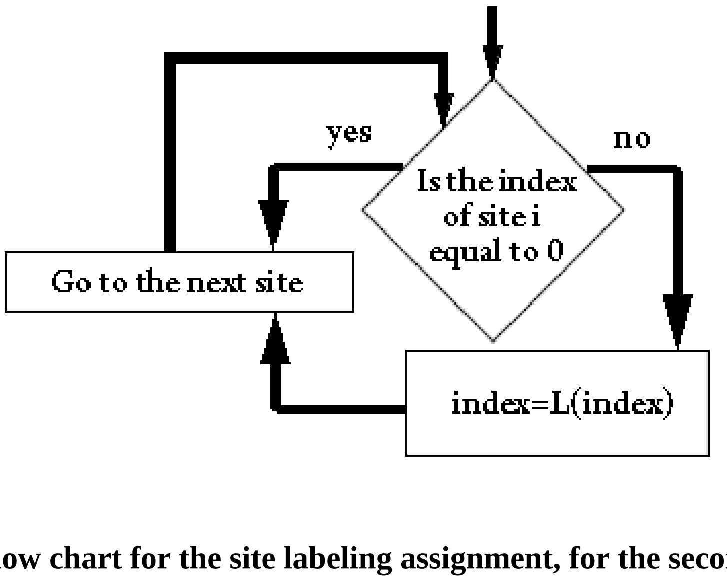

Figure 3: Flow chart for the site labeling assignment, for the second sweep.